Adaptive Control of Non-Minimum Phase Modal Systems using Residual Mode Filters: Part I

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Abstract

Many dynamic systems containing a large number of modes can benefit from adaptive control techniques, which are well suited to applications that have unknown parameters and poorly known operating conditions. In this paper, we focus on a direct adaptive control approach that has been extended to handle adaptive rejection of persistent disturbances. We extend this adaptive control theory to accommodate problematic modal subsystems of a plant that inhibit the adaptive controller by causing the open-loop plant to be non-minimum phase. We will modify the adaptive controller with a Residual Mode Filter (RMF) to compensate for problematic modal subsystems, thereby allowing the system to satisfy the requirements for the adaptive controller to have guaranteed convergence and bounded gains. This paper will be divided into two parts. Here in Part I we will review the basic adaptive control approach and introduce the primary ideas. In Part II, we will present the RMF methodology and complete the proofs of all our results. Also, we will apply the above theoretical results to a simple flexible structure example to illustrate the behavior with and without the residual mode filter.

INTRODUCTION

Applications of control theory to flexible aerospace structures have been many and varied. The survey [13] provides a foundation for structure control with many control approaches and examples. This was based upon a distributed parameter approach to control of flexible structures and other very large-scale systems [14]. Later work created the idea of a Residual Mode Filter (RMF) to offset the destabilizing effect of unmodeled modes in a feedback control environment [15]-[17]. This RMF-based structure control theory has been applied to the complex control

issues for large horizontal-axis utility-sized wind turbines [18]-[21], and is beginning to be applied to aeronautic problems that currently use notch filters, eg for flutter, also we are applying the theory to aircraft control where there are flexible modes in the pilot bandwidth, e.g. large civil tilt rotor.

In this paper, we extend our adaptive control theory [1]-[4], [7] to accommodate modal subsystems of a plant that inhibit the adaptive controller, in particular those residual modes that interfere with the almost strict positive real condition. The systems we consider will be large dimensioned, linear time invariant ones which can be diagonalized or placed into modal form. This will include linear flexible structures of many types. Our adaptive Control approach allows for large dimensioned systems through a foundational use of Ideal Trajectories so that the adaptive controller is of much lower dimension than the plant.

The modification will use the idea of Residual Mode Filters (RMF) introduced for fixed gain controllers in [6]. In this paper the RMF will be used to eliminate the effect of modes that prevent the almost strict positive realness of the overall system by being non-minimum phase. This is a new use of the RMF idea; in previous non-adaptive work the purpose of the RMF was to eliminate or mitigate the destabilizing effect of modes unmodeled in the control system design, whereas here the RMF is applied to reinstate the minimum phase nature of the plant under adaptive control.

Here in Part I we will review the basic adaptive control approach and introduce the primary ideas. In Part II, we will present the RMF methodology and complete the proofs of all our results using results from [8]. Also, we will apply the above theoretical results to a simple flexible structure example to illustrate the behavior with and without the residual mode filter.

Rejection of Persistent Disturbances

The Plant used in this theory section of the paper will be modeled by the linear, time-invariant, finite-dimensional system:

$$\begin{cases} \dot{\mathbf{x}}_p = \mathbf{A}\mathbf{x}_p + \mathbf{B}\mathbf{u}_p + \mathbf{\Gamma}\mathbf{u}_D \\ \mathbf{y}_p = \mathbf{C}\mathbf{x}_p; \ \ \mathbf{x}_p(0) = \mathbf{x}_0 \end{cases} \tag{1}$$

where the *plant state* $\mathbf{x}_p(t)$, is an N_p-dimensional vector, the *control input vector,* $\mathbf{u}_p(t)$, is M-dimensional, and the *sensor output vector*, $\mathbf{y}_p(t)$, is P-dimensional. The *disturbance input vector*, $\mathbf{u}_D(t)$, is M_D-dimensional and will be thought to come from the *Disturbance Generator*:

$$\begin{cases} \mathbf{u}_D = \mathbf{\Theta}\,\mathbf{z}_D \\ \dot{\mathbf{z}}_D = \mathbf{F}\,\mathbf{z}_D;\ \mathbf{z}_D(0) = \mathbf{z}_0 \end{cases} \quad 2)$$

where the *disturbance state*, $\mathbf{z}_D(t)$, is N_D-dimensional. All matrices in (1)-(2) will have the appropriate compatible dimensions. Such descriptions of persistent disturbances were first used in [5] to describe signals of known form but unknown amplitude. Equation (2) can be rewritten as in [3] in a form that is not a dynamical system, which is sometimes easier to use:

$$\begin{cases} \mathbf{u}_D = \mathbf{\Theta}\,\mathbf{z}_D \\ \mathbf{z}_D = \mathbf{L}\phi_D \end{cases} \quad (3)$$

where ϕ_D is a vector composed of the known basis functions for the solution of $\mathbf{u}_D = \mathbf{\Theta}\,\mathbf{z}_D$, i.e., ϕ_D are the basis functions which make up the known form of the disturbance, and $\mathbf{L}$ is a matrix of dimension N_D x dim(ϕ_D). For the analysis performed in this paper, the amplitude of the disturbance does not need to be known, so $(\mathbf{L}, \mathbf{\Theta})$ can be unknown. For a better understanding of the disturbance generator, consider the example of a disturbance generator for a *step disturbance;* in the form of equation (2), a step disturbance would have $\Theta = 1$ and $\mathbf{F} = 0$, in the form of equation (3), a step disturbance would have $\phi_D \equiv 1$.

In [5]-[6], as with much of the control literature, it is assumed that the plant and disturbance generator parameter matrices, $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{\Gamma}, \mathbf{\Theta}, \mathbf{F})$, are known. This knowledge of the plant and its disturbance generator allows the Separation Principle of Linear Control Theory to be invoked to arrive at a State-Estimator based, linear controller which can suppress the persistent disturbances via feedback. In this paper, we will *not* assume that the plant and disturbance generator parameter matrices, $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{\Gamma}, \mathbf{\Theta})$, are known. But, we will assume that the disturbance generator parameter from (2), **F**, is known, i.e., the form of the disturbance functions is known. In many cases, knowledge of **F** is not a severe restriction, since the disturbance function is often of known form but unknown amplitude.

Our control objective will be to cause the output of the plant, $\mathbf{y}_p(t)$, to asymptotically track the output of a known reference model, $\mathbf{y}_m(t)$. The Reference Model is given by

$$\begin{cases} \dot{\mathbf{x}}_m = \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m\mathbf{u}_m;\ \ \mathbf{x}_m(\mathbf{0}) = \mathbf{x}_0^m \\ \mathbf{y}_m = \mathbf{C}_m\mathbf{x}_m \end{cases} \quad (4)$$

where the *reference model state*, $\mathbf{x}_m(t)$, is an N_m-dimensional vector. The *reference model output,* $\mathbf{y}_m(t)$, must have the *same* dimension as the plant output, $\mathbf{y}_p(t)$. The excitation of the reference model is accomplished via the vector, $\mathbf{u}_m(t)$, which is generated by

$$\dot{\mathbf{u}}_m = \mathbf{F}_m \mathbf{u}_m; \;\; \mathbf{u}_m(0) = \mathbf{u}_0^m \tag{5}$$

It is assumed that the reference model is stable and the model parameters, $(\mathbf{A}_m, \mathbf{B}_m, \mathbf{C}_m, \mathbf{F}_m)$, *are known.*

As in [5]-[6], we define the *Ideal Trajectories* for the plant given by (1) as linear combinations of the plant states, the control inputs, and the disturbance inputs:

$$\begin{cases} \mathbf{x}_* = \mathbf{S}_{11}^* \mathbf{x}_m + \mathbf{S}_{12}^* \mathbf{u}_m + \mathbf{S}_{13}^* \mathbf{z}_D \\ \mathbf{u}_* = \mathbf{S}_{21}^* \mathbf{x}_m + \mathbf{S}_{22}^* \mathbf{u}_m + \mathbf{S}_{23}^* \mathbf{z}_D \end{cases} \tag{6}$$

where $\mathbf{x}_*(t)$ is the *ideal trajectory*, $\mathbf{u}_*(t)$ is the *ideal control*, $\mathbf{u}_*(t)$ and

$$\begin{cases} \dot{\mathbf{x}}_* = \mathbf{A}\mathbf{x}_* + \mathbf{B}\mathbf{u}_* + \mathbf{\Gamma}\mathbf{u}_D; \;\; \mathbf{x}_*(0) = \mathbf{x}_0 \\ \mathbf{y}_* = \mathbf{C}\mathbf{x}_* = \mathbf{y}_m \end{cases} \tag{7}$$

Note that the *ideal output*, $\mathbf{y}_*(t)$, matches the reference model output, $\mathbf{y}_m(t)$. If such ideal trajectories exist, they will produce exact output tracking.

By substituting the ideal trajectories given in (6) into (7) and by using the disturbance generator given by (2), the ideal trajectories can be made to match the reference model (4)-(5) with the following *Model Matching Conditions*:

$$\begin{cases} \mathbf{A}\mathbf{S}_{11}^* + \mathbf{B}\mathbf{S}_{21}^* = \mathbf{S}_{11}^* \mathbf{A}_m \\ \mathbf{A}\mathbf{S}_{12}^* + \mathbf{B}\mathbf{S}_{22}^* = \mathbf{S}_{11}^* \mathbf{B}_m + \mathbf{S}_{12}^* \mathbf{F}_m \\ \mathbf{A}\mathbf{S}_{13}^* + \mathbf{B}\mathbf{S}_{23}^* + \mathbf{\Gamma}\mathbf{\Theta} = \mathbf{S}_{13}^* \mathbf{F} \\ \mathbf{C}\mathbf{S}_{11}^* = \mathbf{C}_m \\ \mathbf{C}\mathbf{S}_{12}^* = \mathbf{0} \\ \mathbf{C}\mathbf{S}_{13}^* = \mathbf{0} \end{cases} \tag{8}$$

The model matching conditions given in (8) are necessary and sufficient conditions for the existence of ideal trajectories. Solutions to these matching conditions must exist for later analysis, but explicit solutions need never be known for the adaptive controller design. Necessary and sufficient conditions for the existence and uniqueness of solutions to (8) are given in [9]. We repeat this result here for completeness and the proof is given in the Appendix found in Part II.

Lemma 1: *If CB is nonsingular, there exist unique solutions to the Linear Matching Conditions (8) when* $T(s) \equiv C(sI - A)^{-1}B$ *shares no transmission zeros with the eigenvalues of* A_m, F_m, or F.

The desired control objective is for the output of the plant to asymptotically track the output of the reference model. We define the *output error vector* as:

$$\mathbf{e}_y \equiv \mathbf{y}_p - \mathbf{y}_m \tag{9}$$

To achieve the desired control objective, we want $\mathbf{e}_y \xrightarrow[t\to\infty]{} 0$. We define the *state tracking error* as follows:

$$\mathbf{e}_* \equiv \mathbf{x}_p - \mathbf{x}_* \tag{10}$$

Using (7) and (10), we can write the output error vector as:

$$\mathbf{e}_y \equiv \mathbf{y}_p - \mathbf{y}_m = \mathbf{y}_p - \mathbf{y}_* = \mathbf{C}\mathbf{x}_p - \mathbf{C}\mathbf{x}_* = \mathbf{C}\mathbf{e}_* \tag{11}$$

Furthermore, if we let $\Delta\mathbf{u} \equiv \mathbf{u}_p - \mathbf{u}_*$, from (1) and (7) we have

$$\dot{\mathbf{e}}_* = \mathbf{A}\mathbf{e}_* + \mathbf{B}\Delta\mathbf{u} \tag{12}$$

For analysis purposes, we define a *Fixed Gain Controller*

$$\mathbf{u}_p = \mathbf{u}_* + \mathbf{G}_e^*\mathbf{e}_y \tag{13}$$

If we use the fixed gain control law (13) in the plant given by (1), combined with the definition of $\dot{\mathbf{x}}_*$ from (7) and the output error vector in the form of equation (11), we obtain:

$$\dot{\mathbf{e}}_* = \left(\mathbf{A} + \mathbf{B}\mathbf{G}_e^*\mathbf{C}\right)\mathbf{e}_* \tag{14}$$

We can summarize the above by the following:

Theorem 1: *If* $(\mathbf{A},\mathbf{B},\mathbf{C})$ *is output feedback stabilizable with a gain* $\mathbf{G}_e^*$*, i.e., the eigenvalues of* $\mathbf{A}_C \equiv \mathbf{A}+\mathbf{B}\mathbf{G}_e^*\mathbf{C}$ *are all to the left of the jω-axis, then the fixed gain controller, (13), will produce asymptotic output tracking, i.e.,* $e_y \xrightarrow[t\to\infty]{} 0$.

If all the plant parameters, $(\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{\Gamma},\mathbf{\Theta},\mathbf{F})$, are known, then the fixed gain controller given by (13) with a state estimator for $\mathbf{z}_D$ would be adequate for asymptotic tracking. Note that output feedback stabilization of $(\mathbf{A},\mathbf{B},\mathbf{C})$ can be accomplished when

$$M+P+N_D>N_p \tag{15}$$

and $(\mathbf{A},\mathbf{B},\mathbf{C})$ is controllable and observable [9]. In (13), detailed knowledge of the parameter matrices is not required, suggesting that an adaptive control scheme might be possible under our original assumptions that $(\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{\Gamma},\mathbf{\Theta})$ are unknown and $\mathbf{F}$ from (2) is known.

Consider the plant given by (1) with the disturbance generator given by (3). Our control objective for this system will be accomplished by an *Adaptive Control Law* of the form:

$$\mathbf{u}_p=\mathbf{G}_m\mathbf{x}_m+\mathbf{G}_u\mathbf{u}_m+\mathbf{G}_e\mathbf{e}_y+\mathbf{G}_D\phi_D \tag{16}$$

where $\mathbf{G}_m,\mathbf{G}_u,\mathbf{G}_e,\text{and }\mathbf{G}_D$ are matrices of the appropriate compatible dimensions, whose definitions will be given later. We develop the gain adaptation laws to make asymptotic output tracking possible by first forming the following which are intended to simplify our notation:

$$\begin{cases}\Delta\mathbf{G}_u\equiv\mathbf{G}_u-\mathbf{S}_{22}^*\\ \Delta\mathbf{G}_m\equiv\mathbf{G}_m-\mathbf{S}_{21}^*\\ \Delta\mathbf{G}_e\equiv\mathbf{G}_e-\mathbf{G}_e^*\\ \Delta\mathbf{G}_D\equiv\mathbf{G}_D-\mathbf{S}_{23}^*\mathbf{L}\end{cases} \tag{17}$$

The starred gains in (17) are for analysis and come from the ideal trajectory, $\mathbf{x}_*$, of equation (6) with $\mathbf{z}_D$ in the form given in (3), which is then substituted into the fixed gain controller (13). Using (6), (7), and the adaptive control law (16), we can define:

$$\begin{aligned}\Delta\mathbf{u} &= \mathbf{u}_p - \mathbf{u}_* \\ &= \Delta\mathbf{G}_u\mathbf{u}_m + \Delta\mathbf{G}_m\mathbf{x}_m + \left(\Delta\mathbf{G}_e + \mathbf{G}_e^*\right)\mathbf{e}_y + \Delta\mathbf{G}_D\phi_D\end{aligned} \tag{18}$$

Then, via (11), (12), and (18), with appropriate definitions, we have:

$$\begin{aligned}\dot{\mathbf{e}}_* &= \mathbf{A}\mathbf{e}_* + \mathbf{B}\Delta\mathbf{u} \\ &= \left(\mathbf{A} + \mathbf{B}\mathbf{G}_e^*\mathbf{C}\right)e_* + \mathbf{B}\left[\Delta\mathbf{G}_u \quad \Delta\mathbf{G}_m \quad \Delta\mathbf{G}_e \quad \Delta\mathbf{G}_D\right]\eta \\ &= \mathbf{A}_C\mathbf{e}_* + \mathbf{B}\Delta\mathbf{G}\eta\end{aligned} \tag{19}$$

where $\eta \equiv [\mathbf{u}_m^T \ \mathbf{x}_m^T \ \mathbf{e}_y^T \ \phi_D^T]^T$ is the *vector of available information*. We combine (12) and (19) to obtain the *Tracking Error System*:

$$\begin{cases}\dot{\mathbf{e}}_* = \mathbf{A}_C\mathbf{e}_* + \mathbf{B}\Delta\mathbf{G}\eta \\ \mathbf{e}_y = \mathbf{C}\mathbf{e}_*\end{cases} \tag{20}$$

Now we specify the *Adaptive Gain Laws*:

$$\dot{\mathbf{G}} = -\mathbf{e}_y\eta^T\mathbf{H} \tag{21}$$

where $\mathbf{H} = [\mathbf{h}_{ii}]$, *i=1,2,…,4* is an arbitrary, positive definite matrix (i.e., $\mathbf{H} > 0$). This yields

$$\begin{cases}\dot{\mathbf{G}}_u = -\mathbf{e}_y\mathbf{u}_m^T\mathbf{h}_{11} \\ \dot{\mathbf{G}}_m = -\mathbf{e}_y\mathbf{x}_m^T\mathbf{h}_{22} \\ \dot{\mathbf{G}}_e = -\mathbf{e}_y\mathbf{e}_y^T\mathbf{h}_{33} \\ \dot{\mathbf{G}}_D = -\mathbf{e}_y\phi_D^T\mathbf{h}_{44}\end{cases} \tag{22}$$

Our Adaptive Controller is specified by (16) with the above adaptive gain laws (22). Note that none of the starred gains used in the earlier analysis appear in the realizable control law, (16) and (22). Next we will analyze the stability of this controller.

The *closed-loop adaptive system* consists of (1)-(5), (9), (16), and (22). Using (20) and (21), we have

$$\begin{cases} \dot{\mathbf{e}}_* = \mathbf{A}_C \mathbf{e}_* + \mathbf{B}\Delta\mathbf{G}\eta \\ \Delta\dot{\mathbf{G}} = \dot{\mathbf{G}} = -\mathbf{e}_y \eta^T \mathbf{H} \\ \mathbf{e}_y = \mathbf{C}\mathbf{e}_* \end{cases} \tag{23}$$

where $\mathbf{A}_C \equiv \mathbf{A} + \mathbf{B}\mathbf{G}_e^* \mathbf{C}$. We are able to obtain (23) from (21) because $\Delta\mathbf{G} \equiv \mathbf{G} - \mathbf{G}_*$ where $\mathbf{G}_* \equiv [\mathbf{S}_{22}^* \;\; \mathbf{S}_{21}^* \;\; \mathbf{G}_e^* \;\; \mathbf{S}_{23}^* \mathbf{L}]$ is constant (although generally unknown). The stability of the nonlinear system (23) can be analyzed using Lyapunov Theory. We form the positive definite functions:

$$\begin{cases} V_1(\mathbf{e}_*) \equiv \dfrac{1}{2} \mathbf{e}_*^T \mathbf{P} \mathbf{e}_* \\ V_2(\Delta\mathbf{G}) \equiv \dfrac{1}{2} trace(\Delta\mathbf{G}\mathbf{H}^{-1}\Delta\mathbf{G}^T) \end{cases} \tag{24}$$

where $\mathbf{P} > 0$ is the solution of the following pair of equations:

$$\begin{cases} \mathbf{A}_C^T \mathbf{P} + \mathbf{P}\mathbf{A}_C = -\mathbf{Q}; \; \mathbf{Q} > 0 \\ \mathbf{P}\mathbf{B} = \mathbf{C}^T \end{cases} \tag{25}$$

These equations are usually known as the *Kalman-Yacubovic Conditions*. The existence of a symmetric positive definite solution of (25) is known to be equivalent to the following condition:

$$\mathbf{T}_C(s) \equiv \mathbf{C}(s\mathbf{I} - \mathbf{A}_C)^{-1}\mathbf{B} \text{ is strictly positive real } (SPR) \tag{26}$$

For a proof of this equivalence, see [12] App. B. The *strict positive realness* of $\mathbf{T}_C(s)$ means that for some $\sigma > 0$ and for all ω real,

$$\operatorname{Re} \mathbf{T}_C(-\sigma + j\omega) \geq 0 \tag{27}$$

If we calculate the derivatives, $\dot{V}_i$, along the trajectories of (23), we have, using (25), that

$$\begin{cases} \dot{V}_1 = -\frac{1}{2}\mathbf{e}_*^T\mathbf{Q}\mathbf{e}_* + \mathbf{e}_*^T\mathbf{PB}\Delta\mathbf{G}\eta \\ = -\frac{1}{2}\mathbf{e}_*^T\mathbf{Q}\mathbf{e}_* + \mathbf{e}_y^T v \\ v \equiv \Delta\mathbf{G}\eta \end{cases} \tag{28}$$

and

$$\begin{aligned} \dot{V}_2 &= trace(\Delta\dot{\mathbf{G}}\mathbf{H}^{-1}\Delta\mathbf{G}^T) \\ &= trace(-\mathbf{e}_y\eta^T\mathbf{H})(\mathbf{H}^{-1}\Delta\mathbf{G}^T) \\ &= -trace(\mathbf{e}_y v^T) = -trace(\mathbf{e}_y^T v) = -\mathbf{e}_y^T v \end{aligned} \tag{29}$$

We can form $V \equiv V_1 + V_2 \Rightarrow \dot{V} = -\frac{1}{2}\mathbf{e}_*^{\mathbf{T}}\mathbf{Q}\mathbf{e}_*$ with $\dot{V} \leq 0$. Consequently, Lyapunov theory guarantees stability of the zero equilibrium point of (23) and all trajectories of (24) will remain bounded. This guarantees that both $\mathbf{e}_*$ and $\Delta\mathbf{G}$ are bounded.

We can summarize the above by the following *Closed-Loop Stability Result:*

Theorem 2: Suppose the following are true:

(1) All $\mathbf{u}_m(t)$ are bounded (i.e., all eigenvalues of $\mathbf{F}_m$ are in the closed left-half plane);

(2) The reference model (4) is stable (i.e., all eigenvalues of $\mathbf{A}_m$ are in the open left-half plane);

(3) ϕ_D is bounded (i.e., all eigenvalues of $\mathbf{F}$ are in the closed left-half plane and any eigenvalues on the imaginary axis are simple);

(4) ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$) is Almost Strict Positive Real (ASPR), i.e., $\mathbf{T}_C(s) \equiv \mathbf{C}(s\mathbf{I} - \mathbf{A}_C)^{-1}\mathbf{B}$ is strictly positive real.

Then $\mathbf{e}_*$ and $\Delta\mathbf{G}$ are bounded and $\mathbf{e}_* \xrightarrow[t\to\infty]{} 0$ and $\mathbf{e}_y \equiv \mathbf{y}_p - \mathbf{y}_m = \mathbf{C}\mathbf{e}_* \xrightarrow[t\to\infty]{} 0$.

See the Appendix in Part II for a proof of Theorem 2.

This stability analysis shows that asymptotic tracking occurs and the adaptive gains remain bounded. It does *not* prove that $\Delta\mathbf{G} \xrightarrow[t\to\infty]{} \mathbf{0}$. In fact, the gain adaptation laws (22) may not converge to the starred gains in (8); however, this is not required for the adaptive controller to achieve its goals.

Conclusions for Part I

We have reviewed our adaptive control theory here. This theory accounts for adaptive model tracking and for leakage of the disturbance term into the Q modes. However, the results require that the error system be minimum phase. In Part II, we will show how to modify the adaptive control with residual mode filters to deal with non-minimum phase systems.

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Adaptive Control of Non-Minimum Phase Modal Systems using Residual Mode Filters: Part II

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Abstract

In Part II, we extend our adaptive control theory to accommodate problematic modal subsystems of a plant that inhibit the adaptive controller by causing the open-loop plant to be non-minimum phase. We will modify the adaptive controller with a Residual Mode Filter (RMF) to compensate for problematic modal subsystems, thereby allowing the system to satisfy the requirements for the adaptive controller to have guaranteed convergence and bounded gains. Also, we will apply the above theoretical results to a simple flexible structure example to illustrate the behavior with and without the residual mode filter.

INTRODUCTION

In Part II, we continue the development of the adaptive control approach. We will keep the consecutive equation numbering from Part I as well as the same reference list. We modify the adaptive control using the idea of Residual Mode Filters (RMF) introduced for fixed gain controllers in [6]. In this paper the RMF will be used to eliminate the effect of modes that prevent the almost strict positive realness (ASPR) of the overall system by being non-minimum phase. We have maintained the same reference list here as in the previous paper to make both papers more readable.

Residual Mode Filter Augmentation of Adaptive Controller

In some cases the plant in (1) does not satisfy the requirements of ASPR. Instead, there maybe be a modal subsystem that inhibits this property. This section will present new results for our adaptive control theory. We will modify the adaptive controller with a Residual Mode Filter (RMF) to compensate for the troublesome modal subsystem, or the Q modes, as was done in [6] for fixed gain non-adaptive controllers. Here we present the theory for adaptive controllers modified by RMFs. In a previous paper, we examined the RMF with adaptive control, but assumed that there was no leakage of the disturbance into the Q modes [7]. Here we will deal with the issue of disturbances propagating through these modes.

Let us assume that (1) can be partitioned into the following modal form:

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{x}_Q \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A_Q \end{bmatrix} \begin{bmatrix} x \\ x_Q \end{bmatrix} + \begin{bmatrix} B \\ B_Q \end{bmatrix} u_p + \begin{bmatrix} \Gamma \\ \varepsilon\Gamma_Q \end{bmatrix} u_D \\ y_p = \begin{bmatrix} C & C_Q \end{bmatrix} \begin{bmatrix} x \\ x_Q \end{bmatrix}; \varepsilon \geq 0 \end{cases} \tag{28}$$

Define $x_p \equiv \begin{bmatrix} x \\ x_Q \end{bmatrix}$, $A_p = \begin{bmatrix} A & 0 \\ 0 & A_Q \end{bmatrix}$, $B_p = \begin{bmatrix} B \\ B_Q \end{bmatrix}$, $\Gamma_p = \begin{bmatrix} \Gamma \\ \varepsilon\Gamma_Q \end{bmatrix}$, $C_p^T = \begin{bmatrix} C \\ C_Q \end{bmatrix}$, and

Disturbance Generator $\begin{cases} \dot{z}_D = F z_D \\ u_D = \theta z_D \end{cases}$ or $z_D = L\phi_D$ as before in (2)-(3).

The Output Tracking Error and control objective remain as in (4)-(5), *i.e.* $e_y \equiv y_p - y_m \xrightarrow[t\to\infty]{} 0$.

However, now we will only assume that the subsystem (A,B,C) is Almost Strictly Positive Real, rather than the full un-partitioned plant (A_p,B_p,C_p), and the modal subsystem (A_Q,B_Q,C_Q) will be *known and open-loop stable, i.e.,* A_Q is stable. Also note that this subsystem is directly affected by the disturbance input. Recall that ASPR means $CB > 0$ and $P(s) = C(sI - A)^{-1}B$ is minimum phase. So, in summary, the actual plant has an ASPR subsystem and a known modal subsystem that is stable but inhibits the property of ASPR for the full plant. Hence, this modal subsystem must be compensated or filtered away.

We define the *Residual Mode Filter* (RMF):

$$\begin{cases} \dot{\hat{x}}_Q = A_Q \hat{x}_Q + B_Q u_p \\ \hat{y}_Q = C_Q \hat{x}_Q \end{cases} \tag{29}$$

And the *compensated tracking error*:

$$\tilde{e}_y \equiv e_y - \hat{y}_Q \tag{30}$$

Now we let $e_Q \equiv \hat{x}_Q - x_Q$ and obtain:

$$\dot{e}_Q = A_Q e_Q - \varepsilon \Gamma_Q u_D \tag{31}$$

Consequently,

$$\begin{aligned} \tilde{e}_y &\equiv e_y - \hat{y}_Q = C\Delta x + C_Q x_Q - [C_Q x_Q + C_Q e_Q] \\ &= C\Delta x - C_Q e_Q \end{aligned} \tag{32}$$

As in [1]-[2], we define the Ideal Trajectories, but only for the ASPR Subsystem:

$$\begin{cases} \dot{x}_* = Ax_* + Bu_* + \Gamma u_D \\ y_* = Cx_* = 0 \end{cases} \tag{33}$$

with $\begin{cases} x_* = S_1^* z_D \\ u_* = S_2^* z_D \end{cases}$. This is equivalent to the *Matching Conditions:*

$$\begin{cases} S_1^* F = AS_1^* + BS_2^* + \Gamma\theta \\ CS_1^* = 0 \end{cases} \tag{34}$$

which are known to be uniquely solvable when *CB* is nonsingular. However, we do *not* need to know the actual solutions for this adaptive control approach.

Let $\begin{cases} \Delta x \equiv x - x_* \\ \Delta u \equiv u_p - u_* \\ \Delta\tilde{y} \equiv \tilde{e}_y = C\Delta x - C_Q e_Q \end{cases}$. Then we have

$$\begin{cases} \Delta\dot{x} = A\Delta x + B\Delta u \\ \Delta\tilde{y} = C\Delta x - C_Q e_Q \end{cases} \tag{35}$$

because, from (33), $y_* = 0$. This system can be rewritten:

$$\begin{cases} \begin{bmatrix} \Delta\dot{x} \\ \dot{e}_Q \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A_Q \end{bmatrix} \begin{bmatrix} \Delta x \\ e_Q \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta u = \overline{A} \begin{bmatrix} \Delta x \\ e_Q \end{bmatrix} + \overline{B}\Delta u + \varepsilon\overline{\Gamma}_Q u_D \\ \Delta\widetilde{y} = \begin{bmatrix} C & -C_Q \end{bmatrix} \begin{bmatrix} \Delta x \\ e_Q \end{bmatrix} = \overline{C} \begin{bmatrix} \Delta x \\ e_Q \end{bmatrix} \end{cases} \tag{36}$$

Now we have the following:

Lemma: $\left(\overline{A} = \begin{bmatrix} A & 0 \\ 0 & A_Q \end{bmatrix}, \overline{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \overline{C} = \begin{bmatrix} C & -C_Q \end{bmatrix} \right)$ is ASPR if and only if $\left(A, B, C\right)$ is ASPR.

Proof: $\overline{C}\overline{B} = \begin{bmatrix} C & -C_Q \end{bmatrix} \begin{bmatrix} B \\ 0 \end{bmatrix} = CB > 0$ and

$$\begin{aligned} \overline{P}(s) &\equiv \overline{C}(sI - \overline{A})^{-1}\overline{B} \\ &= \begin{bmatrix} C & -C_Q \end{bmatrix} \begin{bmatrix} (sI - A)^{-1} & 0 \\ 0 & (sI - A_Q)^{-1} \end{bmatrix} \begin{bmatrix} B \\ 0 \end{bmatrix} \\ &= C(sI - A)^{-1}B = P(s) \end{aligned}$$

is minimum phase. End of proof.

So there exists G_e^* such that $(\overline{A}_C \equiv \overline{A} + \overline{B}G_e^*\overline{C}, \overline{B}, \overline{C})$ is Strictly Positive Real (SPR) when (A, B, C) is ASPR. Consequently, as is well known from the Kalman-Yacubovic Theorem, there exists $\overline{P}, \overline{Q} > 0$ such that

$$\begin{cases} \overline{A}_C^T \overline{P} + \overline{P}\overline{A}_C = -\overline{Q} \\ \overline{P}\overline{B} = \overline{C}^T \end{cases} \tag{37}$$

We now write the *modified adaptive control law with RMF*:

$$\begin{cases} u_p \equiv G_u u_m + G_m x_m + G_e \widetilde{e}_y + G_D \phi_D \\ \widetilde{e}_y \equiv y_p - \hat{y}_Q \\ \dot{\hat{x}}_Q = A_Q \hat{x}_Q + B_Q u_p \\ \hat{y}_Q = C_Q \hat{x}_Q \end{cases} \tag{38}$$

with modified adaptive gains given by

$$\begin{cases} \dot{G}_u = -\tilde{e}_y u_m h_u; h_u > 0 \\ \dot{G}_m = -\tilde{e}_y x_m h_m; h_m > 0 \\ \dot{G}_e = -\tilde{e}_y \tilde{e}_y^T h_e; h_e > 0 \\ \dot{G}_D = -\tilde{e}_y \phi_D^T h_D; h_D > 0 \end{cases} \tag{39}$$

Finally, we have the following stability result:

Theorem 3: In (9), let (A,B,C) ASPR, A_Q stable, ϕ_D bounded. Then the Modified Adaptive Controller with RMF in (19)-(20) produces $e_y = y_p$ and e_Q ultimately bounded into a ball of radius $R_* \equiv \varepsilon \frac{\left(1+\sqrt{p_{\max}}\right)}{a\sqrt{p_{\min}}} M_v$ with exponential rate and bounded adaptive gains (G_e, G_D).

Proof: From (19), we have $u_p \equiv G_e \tilde{e}_y + G_D \phi_D$, so we can write

$$\begin{aligned} \Delta u &\equiv u_p - u_* \\ &= [G_e \tilde{e}_y + G_D \phi_D] - [S_2^* L]\phi_D \\ &= G_e^* \tilde{e}_y + \Delta G \eta \end{aligned}$$

where $\begin{cases} \Delta G_e \equiv G_e - G_e^* \\ \Delta G_D \equiv G_D - (S_2^* L) \\ \Delta G \equiv G - G_* = \begin{bmatrix} \Delta G_e & \Delta G_D \end{bmatrix} \\ \eta \equiv \begin{bmatrix} \tilde{e}_y \\ \phi_D \end{bmatrix} \end{cases}$. Then

$$\begin{cases} \dot{\zeta} = \overline{A}\zeta + \overline{B}\Delta u = \overline{A}_C \zeta + \overline{B} w + \varepsilon \overline{\Gamma}_Q u_D \\ \tilde{e}_y = \overline{C}\zeta \end{cases} \tag{40}$$

with $\zeta \equiv \begin{bmatrix} \Delta x \\ e_Q \end{bmatrix}$, $w \equiv \Delta G \eta$, $\overline{A}_C \equiv \overline{A} + \overline{B} G_e^* \overline{C}$.

From (20), we can see that

$$\dot{G} = \Delta \dot{G} = -\tilde{e}_y \eta^T h; h \equiv \begin{bmatrix} h_e & 0 \\ 0 & h_D \end{bmatrix} > 0 \tag{41}$$

Since (A,B,C) is ASPR, and by the lemma, so is $(\overline{A},\overline{B},\overline{C})$, we can we can use the following result from [8] where $\nu \equiv \overline{\Gamma}_Q u_D$ is bounded because the disturbance $u_D = L\phi_D$ is bounded.

Result: *Consider the nonlinear, coupled system of differential equations,*

$$\begin{cases} \dot{\varsigma} = \overline{A}_c \varsigma + \overline{B}\left(G(t) - G^*\right)\eta + \varepsilon\nu \\ \tilde{e}_y = \overline{C}\varsigma \\ \dot{G}(t) = -\tilde{e}_y \eta^{\mathrm{T}} h - aG(t) \end{cases} \tag{42}$$

where G^* is any constant matrix and h is any positive definite constant matrix, each of appropriate dimension. Assume the following:
i) the triple $(\overline{A},\overline{B},\overline{C})$ is SPR,
ii) there exists $M_K > 0$ such that $\left\| \left(G^*\right)^T G^* \right\| \le M_K$, using the trace norm,
iii) there exists $M_\nu > 0$ such that $\sup_{t\ge 0}\left\|\nu(t)\right\| \le M_\nu$,
iv) there exists $a > 0$ such that $a \le \frac{q_{\min}}{2p_{\max}}$, and
v) h satisfies $\left\|h^{-1}\right\|_2 \le \left(\frac{\varepsilon M_\nu}{aM_K}\right)^2$, where p_{min}, p_{max} are the minimum and maximum eigenvalues of $\overline{P}$ and q_{min} is the minimum eigenvalue of $\overline{Q}$ in the system $\begin{cases} \overline{A}_C^T \overline{P} + \overline{P}\overline{A}_C = -\overline{Q} \\ \overline{P}\overline{B} = \overline{C}^T \end{cases}$.
Then the matrix G(t) is bounded and the state $\zeta(t)$ exponentially approaches the ball of radius
$R_* \equiv \varepsilon \frac{\left(1+\sqrt{p_{\max}}\right)}{a\sqrt{p_{\min}}} M_\nu$ with $\varepsilon > 0$.

From this result, we have ζ is ultimately bounded into the ball of radius R_*, which leads to $e_y \equiv y_p = y_p - y_* = C\Delta$ and e_Q ultimately bounded as well. Therefore $G = G_* + \Delta G$ is bounded, as desired. #

Consequently, the radius of the error ball $R_* \equiv \varepsilon \frac{\left(1+\sqrt{p_{\max}}\right)}{a\sqrt{p_{\min}}} M_\nu$ is determined by the size of ε, which is related to the amount of disturbance leakage into

the Q modes. It can be seen that, when there is no leakage of the disturbance into the Q modes ($\varepsilon = 0$), the convergence is asymptotic to zero.

Also, when $\Gamma = B$ and $\Gamma_Q = B_Q$, it is possible to choose $S_1^* = 0$ and $S_2^* = -\theta$ in (34). Then, even if $\varepsilon = 1$, the tracking error will asymptotically go to zero.

Simulation Results with RMF

In this section we will apply the above theoretical results to a very simple flexible structure example to illustrate the behavior with and without the Residual Mode Filter. The structure has a rigid body mode and two flexible modes given by:

$$P(s) = \frac{1+s}{s^2} - \frac{3}{s^2+s+1} + \frac{1}{s^2+s+2} = \frac{s^5+s^4+3s^3+0s^2+3s+1}{s^6+2s^5+4s^4+3s^3+2s^2}.$$

This example can obviously be extended to have many more flexible modes. But we are only trying to illustrate the value of the RMF approach. More seriously complex flexible structures are being addressed but will have to await future papers.

This plant has two non-minimum phase zeros at $0.422 \pm 0.9543j$ and thus does not meet the ASPR condition. However, when the middle mode

$$P_Q(s) = -\frac{s}{s^2+s+1}$$

is removed, the plant becomes: $P(s) = \dfrac{1+s}{s^2} + \dfrac{1}{s^2+s+2} = \dfrac{s^3+3s^2+3s+2}{s^4+s^3+2s^2}$

which is minimum phase and has a state space realization given by:

$$\begin{cases} \dot{x}_p = Ax_p + B(u_p + u_D) \\ y_p = Cx_p \end{cases} \text{ with } A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & -1 \end{bmatrix}, \ \Gamma = B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \ C^T = \begin{bmatrix} 2 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

with $CB = 1$, so CB is nonsingular. Therefore, (A,B,C) is ASPR.

The reference model to be tracked is

$$\begin{cases} \dot{x}_m = (-1)x_m + (1)u_m \\ y_m = (1)x_m \end{cases}$$

which is excited by steps generated by $\dot{u}_m = (0)u_m$. The matching conditions are known to be solvable, but their solution is not needed to apply the theory.

The RMF generated by $P_Q(s) = -\dfrac{3}{s^2+s+1}$ is represented by $A_Q = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$, $\Gamma_Q = B_Q = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_Q = \begin{bmatrix} -3 & 0 \end{bmatrix}$. And we see that $C_Q B_Q = 0$.

The adaptive controller given by (38) - (39) is implemented with h_u=10, h_m=1, h_e=10, h_D=100, and a=0. The disturbance is a nondimensional step of size 10. Setting ε=1, we obtain figs. 1 and 2 from a MatLab/Simulink simulation. The output tracking error is shown to converge to zero in fig. 1. The adaptive gains also converge in fig. 2. This illustrates the behavior of the adaptive controller plus the second order RMF. Without the RMF, the plant and adaptive controller are immediately unstable in closed-loop.

Conclusions

We have proposed a modified adaptive controller with a residual mode filter. The RMF is used to accommodate problematic modes in the system that inhibit the adaptive controller, in particular the ASPR condition. This new theory accounts for adaptive model tracking and for leakage of the disturbance term into the Q modes. A simple three mode example shows that the RMF can restore stability to an otherwise unstable adaptively controlled system. This is done without seriously modifying the adaptive controller design.

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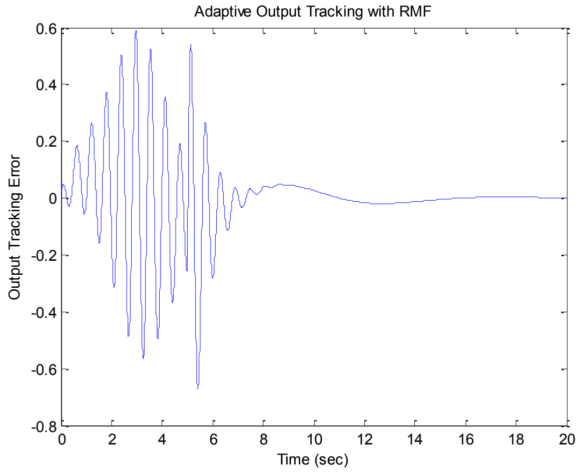

Fig. 1. Nondimensional output tracking response with adaptive controller augmented with RMF.

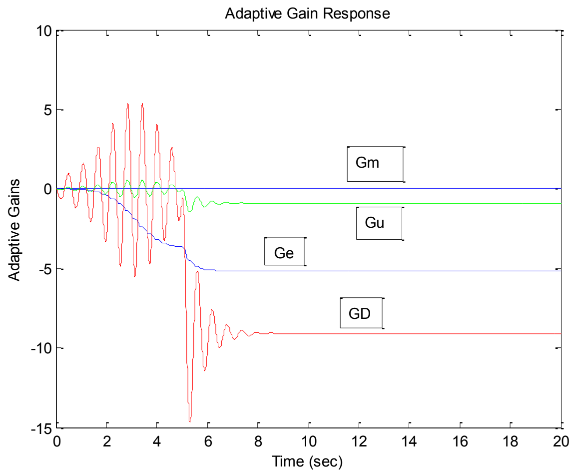

Fig. 2. Adaptive gains, Ge=error gain, Gd=disturbance gain.

Appendix

Proof of **Lemma 1**: The Linear Matching Conditions (8) can be rewritten:

$$\begin{cases} AS_1 + BS_2 = S_1 L_m + H_1 \\ CS_1 = H_2 \end{cases}$$

where $S_1 \equiv \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \end{bmatrix}$, $S_2 \equiv \begin{bmatrix} S_{21}^* & S_{22}^* & S_{23}^* \end{bmatrix}$, $L_m \equiv \begin{bmatrix} A_m & B_m & 0 \\ 0 & F_m & 0 \\ 0 & 0 & F \end{bmatrix}$,

and $\begin{cases} H_1 \equiv \begin{bmatrix} 0 & 0 & -\Gamma\theta \end{bmatrix} \\ H_2 \equiv \begin{bmatrix} C_m & 0 & 0 \end{bmatrix} \end{cases}$.

Suppose CB is nonsingular. Use the coordinate transformation W from Lemma 2 in [11] to put (A, B, C) into *normal form:*

$$\begin{cases} \dot{y} = \bar{A}_{11} y + \bar{A}_{12} z_2 + CBu \\ \dot{z}_2 = \bar{A}_{21} y + \bar{A}_{22} z_2 \end{cases}$$

i.e., there exists $W \equiv \begin{bmatrix} C \\ W_2^T P_2 \end{bmatrix}$ such that $WAW^{-1} \equiv \bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$, $WB = \begin{bmatrix} CB \\ 0 \end{bmatrix} \equiv \bar{B}$, and $CW^{-1} = \begin{bmatrix} I_m & 0 \end{bmatrix} \equiv \bar{C}$ which implies that

$$\begin{aligned} \bar{S}_1 L_m &= WS_1 L_m = WAW^{-1} WS_1 + WBS_2 - WH_1 = \bar{A}\bar{S}_1 + \bar{B} S_2 - \bar{H}_1 \\ &= \bar{A}\bar{S}_1 + \begin{bmatrix} CB \\ 0 \end{bmatrix} S_2 - \bar{H}_1 \end{aligned}$$

and $H_2 = CW^{-1} WS_1 = \bar{C}\bar{S}_1 = \begin{bmatrix} I & 0 \end{bmatrix} \bar{S}_1 = \bar{S}_a$ where $\bar{S}_1 \equiv WS_1 = \begin{bmatrix} \bar{S}_a \\ \bar{S}_b \end{bmatrix}$. From this we have that $\begin{bmatrix} H_2 \\ \bar{S}_b \end{bmatrix} L_m = \bar{A} \begin{bmatrix} H_2 \\ \bar{S}_b \end{bmatrix} + \begin{bmatrix} CB \\ 0 \end{bmatrix} S_2 - \begin{bmatrix} \bar{H}_a \\ \bar{H}_b \end{bmatrix}$ which implies that

$$\begin{cases} S_2 = (CB)^{-1} [H_2 L_m + \bar{H}_a - (\bar{A}_{11} H_2 + \bar{A}_{12} \bar{S}_b)] \\ \bar{S}_b L_m = \bar{A}_{22} \bar{S}_b + (\bar{A}_{21} H_2 - \bar{H}_b) \end{cases}.$$

Now, if $(\bar{A}_{22}, L_m)$ share no eigenvalues, it is well known [5] that we can solve the above for a unique $\bar{S}_b$ and conversely, then $\bar{S}_1 = \begin{bmatrix} H_2 \\ \bar{S}_b \end{bmatrix}$, $S_2 = (CB)^{-1} [H_2 L_m + \bar{H}_a - (\bar{A}_{11} H_2 + \bar{A}_{12} \bar{S}_b)]$ and $\bar{A}\bar{S}_b - \bar{S}_b L_m = \bar{H}_b$. Since $(\bar{A}_{22}, L_m)$ share no eigenvalues, this is the same as $\bar{A}_{22}$ sharing no eigenvalues with A_m, F_m or F. But the eigenvalues of $\bar{A}_{22}$ from its normal form are known to be the transmission zeros of the open-loop system (A, B, C); see e.g. [13]. Thus, we have proved the result. End of Proof.

Proof of **Theorem 2**

It was already shown that e_* and ΔG are bounded. To prove that $e_* \xrightarrow[t\to\infty]{} 0$, we must use the following version of Barbalat's lemma; see [19] pp. 210-211:

Lemma: If *f(t)* is a real, differentiable function on $(0,\infty)$ with $\lim_{t\to\infty} f(t)$ finite and $\frac{df}{dt}$ uniformly continuous, then $\lim_{t\to\infty} \frac{df}{dt} = 0$.

We have already seen that $\dot{V}(t) \le 0$; therefore $V(t) - V(0) = \int_0^t \dot{V}(\tau) d\tau \le 0$ or $0 \le V(t) \le V(0)$ where $V(0) < \infty$. Hence $\lim_{t\to\infty} V(t)$ is finite. Also, $\dot{V}(t)$ is bounded because

$$
\begin{aligned}
\ddot{V}(t) &= -\left(e_*^T Q \dot{e}_*\right) \le \|e_*\|\|Q\|\|\dot{e}_*\| \\
&= \|e_*\|\|Q\|\|A_C e_* + B\Delta G \eta\| \\
&\le \|e_*\|\|Q\|\left(\|A_C\|\|e_*\| + \|B\|\|\Delta G\|\|\eta\|\right)
\end{aligned}
$$

and e_* and ΔG are bounded by the previous argument via Lyapunov theory. Also η is bounded since u_m is bounded, A_m is stable, $e_y = Ce_*$ is bounded, and ϕ_D is bounded. Thus $\dot{V}(t) = \int_0^t \ddot{V}(\tau) d\tau$ is uniformly continuous and Barbalat's Lemma may be applied to yield:

$0 = \lim_{t\to\infty} \dot{V}(t) = -\lim_{t\to\infty}\left(e_*^T Q e_*\right)$. Since $Q > 0$, we have $e_* \xrightarrow[t\to\infty]{} 0$, as desired.

End of Proof.